Design considerations for active membrane mirrors

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Membrane mirrors on a finite number of supports are limited by "print-through" of the actuators.

There exists a straightforward relationship between the membrane thickness, the errors that need to be corrected, and the magnitude of the print through.

Effect of using actuators to correct low order shape errors

- *Actuators must apply some force to the membrane to correct its shape*

  CTE variations in the membrane will cause the surface to warp when cooled

  Temperature variations will cause the membrane to warp

  The surface will have residual fabrication errors from polishing and support

- Force applied by the actuators will cause local "bumps" that are calculated as

$$\delta_{rms} = 0.0012 \frac{q}{D} \left(\frac{A}{N} \right)^2$$

D = modulus of rigidity $= E\, t^3/12(1\text{-}\nu^2)$

q = Force per unit area applied by actuators

From Nelson, et al, Proc. SPIE 332 (1982).

Calculation of reaction pressure to hold membrane figure

If a region on the membrane expands, it pushes against the area around it and makes a "blister"

The relationship between pressure to contain his blister and the internal stress is calculated assuming static equilibrium

membrane stress σ

t

normal force/area q

R
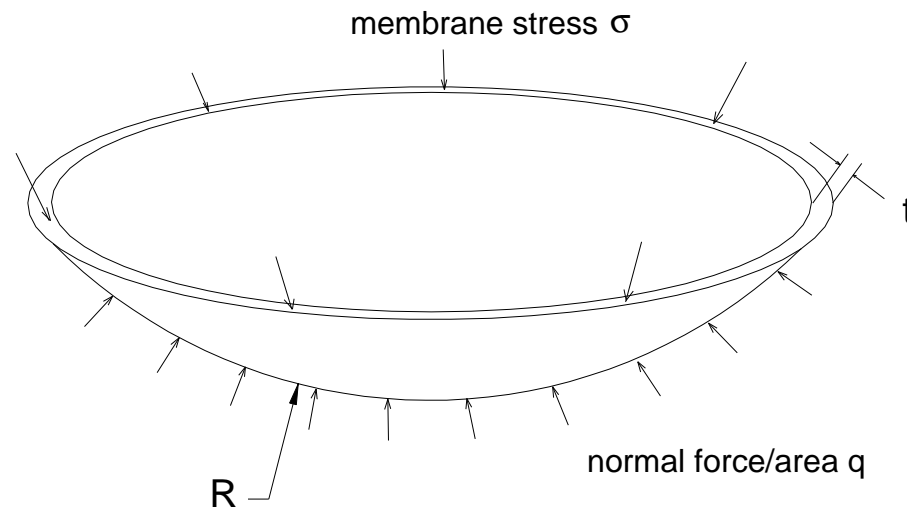
Reaction pressure q = 2 t σ/ R

Calculation of figure error due to CTE, temperature variations

If edge of blister is unyielding, replace σ with E Δ(α T), but this is unrealistic

Better modeled using Δ(α T) with sinusoidal or Gaussian random spatial profile. We used finite element modeling empirically determine σ ~ 0.36 E Δ(α T) for R/t of ~10^4

We use this to calculate figure due to support forces holding the correct global shape

$$\delta_{rms} = \frac{0.01(1-\nu^2)}{R \cdot t^2 \cdot \left(\dfrac{N}{A}\right)^2} \cdot \Delta(\alpha T)$$

Interesting results from analysis

- These effects are independent of the spatial scale of the stress.

- Using triangular load spreaders to distribute the force from each actuator to 3 points can improve the figure 9 x

- Another form of equation is

$$d_{rms} = \frac{0.01 \cdot \left(1 - n^2\right) \cdot r^2}{R \cdot \left(\dfrac{m}{A}\right)^2 \cdot \left(\dfrac{N}{A}\right)^2} \cdot \Delta(aT)$$

    where m/A is allotted mass per unit area of glass

Does not depend on E!!!!   Rubber is as good as beryllium

Does not depend on spatial scale of error as long as >> actuator spacing

Stiffer materials require more force per actuator to hold their shape, but have proportionally smaller bumps/unit force

Important parameters are density, CTE variations, and temperature variations coupled with at-temperature CTE